Two-Way ANOVA with Repeated Measures on a Factor

Table 12.21: Weights of Anorexic Girls, in Pounds, Before and After Receiving One of Three Treatments

Cognitive Behavioral		Family Therapy		Control	
Weight Before	Weight After	Weight Before	Weight After	Weight Before	Weight After
80.5	82.2	83.8	95.2	80.7	80.2
84.9	85.6	83.3	94.3	89.4	80.1
81.5	81.4	86.0	91.5	91.8	86.4
82.6	81.9	82.5	91.9	74.0	86.3
79.9	76.4	86.7	100.3	78.1	76.1
88.7	103.6	79.6	76.7	88.3	78.1
94.9	98.4	76.9	76.8	87.3	75.1
76.3	93.4	94.2	101.6	75.1	86.7
81.0	73.4	73.4	94.9	80.6	73.5
80.5	82.1	80.5	75.2	78.4	84.6
85.0	96.7	81.6	77.8	77.6	77.4
89.2	95.3	82.1	95.5	88.7	79.5
81.3	82.4	77.6	90.7	81.3	89.6
76.5	72.5	83.5	92.5	78.1	81.4
70.0	90.9	89.9	93.8	70.5	81.8
80.4	71.3	86.0	91.7	77.3	77.3
83.3	85.4	87.3	98.0	85.2	84.2
83.0	81.6			86.0	75.4
87.7	89.1			84.1	79.5
84.2	83.9			79.7	73.0
86.4	82.7			85.5	88.3
76.5	75.7			84.4	84.7
80.2	82.6			79.6	81.4
87.8	100.4			77.5	81.2
83.3	85.2			72.3	88.2
79.7	83.6			89.0	78.8
84.5	84.6				
80.8	96.2				
87.4	86.7				

Source: Thanks to Prof. Brian Everitt, Institute of Psychiatry, London, for these data.

three groups of girls, represented by three independent samples. The second, 'Time,' consists of the two times for observations, (before, after). Each time has the same subjects, so the samples at its levels are dependent. Time is called a ***within-subjects factor***, because comparisons of its categories use repeated measurements on subjects. Treatment is called a ***between-subjects factor***, because comparisons of its categories use different subjects.

Although the two factors (treatment and time) are fixed effects, the analysis differs from ordinary two-way ANOVA. This is because the repeated measurements on the within-subjects factor (time) creates a third effect, a random effect for subjects. Each subject is measured at every category of time. Subjects are said to be ***crossed*** with the time factor. Each subject occurs at only one category of the between subjects factor (treatment). Subjects are said to be ***nested*** within the treatment factor.

As in ordinary two-way ANOVA, we can test each main effect as well as their interaction. However, tests about the within-subjects factor (both its main effect and its interaction with the other fixed effect) use a different error term than the test about the between-subjects main effect. The ordinary sum of squared error term is partitioned into two parts. One part uses the variability between mean scores of subjects. It forms an error term for testing the between-subjects factor. The other part is based on how the pattern of within-subject scores varies among subjects. It forms an error term for any test involving the within-subjects factor.

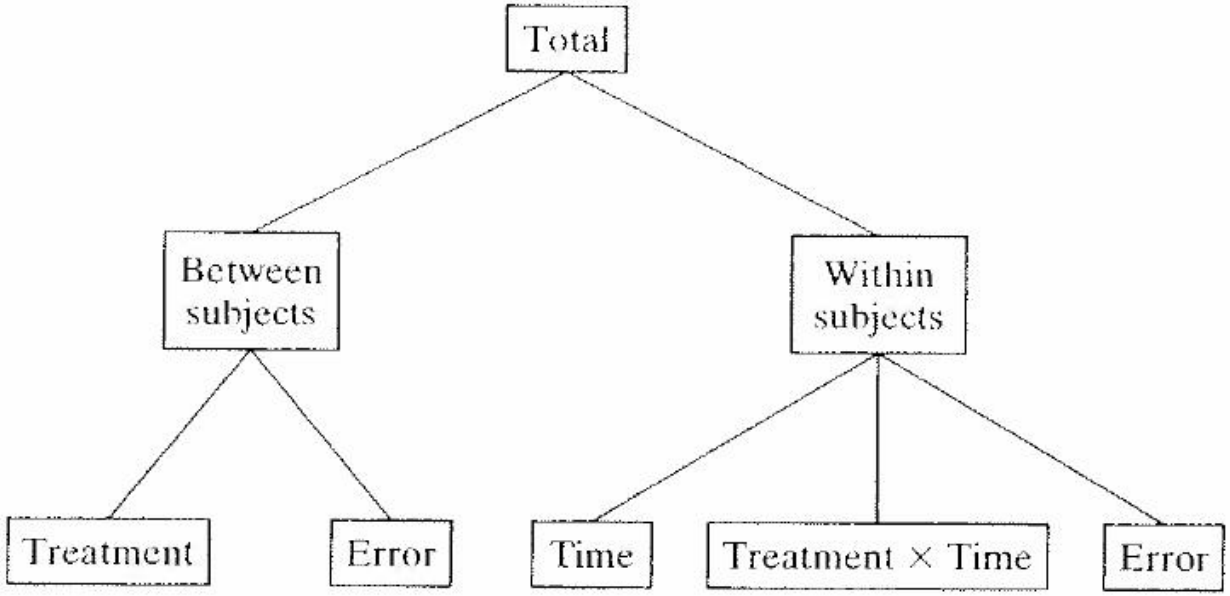


FIGURE 12.6: Partitioning of Variability in Two-Way ANOVA with Treatment and Time Factors and Repeated Measures on Time. Tests involving the within-subjects factor (time) use a separate error term.

Repeated Measures at More than Two Times

In Example 12.8 the repeated measurements occur at two times. When observations are made at several times, the repeated measures ANOVA is more complex. In particular, the results depend on assumptions about the correlation structure of the repeated measurements. The standard test for the within-subject effect assumes *sphericity*, as in 1-way repeated measures ANOVA. Tests of the between-subjects effects are not affected by violation of the sphericity assumption, so no adjustment is needed for that F test.

Alternative *multivariate* ANOVA approaches to testing the within-subjects effects (such as the *Wilks lambda likelihood-ratio test*) make fewer assumptions but often have weaker power. Other recently developed methods allow more varied types of random effects modeling of the correlation structure for the repeated responses, providing options other than sphericity. We'll discuss such approaches in Section 16.1.

More Complex Repeated Measures Analyses

Two-way ANOVA with repeated measures on a factor extends to more complex designs. Suppose, for example, that a study has three factors, A, B, and C, with repeated measures on C. That is, subjects are crossed with C but nested within combinations of levels of A and B. The between-subjects effects, namely, the A and B main effects and the A×B interaction, are tested with a mean square error based on variability between subjects. All effects involving the within-subjects factor C, namely the C main effect, the A×C interaction, the B×C interaction, and the A×B×C interaction, are tested with a separate mean square error.

In some studies with two fixed effects, repeated measures occur on both factors. For instance, we may observe the same subjects for each treatment at each of several times. Then, subjects (a random effect) are crossed with both factors (fixed effects), and an observation occurs for every subject at every combination of factor levels. As in ordinary two-way ANOVA, the effects of interest refer to the fixed effects—their main effects and interaction. The complicating factor is that each test requires a separate mean square error, but software can easily conduct this analysis.

```
'data.frame':   144 obs. of  4 variables:
 $ id   : int  1 2 3 4 5 6 7 8 9 10 ...
 $ trat : Factor w/ 3 levels "C","CB","FT": 2 2 2 ...
 $ y    : num  80.5 84.9 81.5 82.6 79.9 88.7 94.9 ...
 $ tempo: Factor w/ 2 levels "dopo","prima": 2 2 2...
```

```
> fit <- aov(y ~ tempo*trat + Error(subj/tempo))
```

```
> summary(fit) # Table 12.23 Agresti (a parte le SSQ
type III)

Error: subj
          Df Sum Sq Mean Sq F value  Pr(>F)
trat       2    644   322.1   6.201 0.00334 **
Residuals 69   3584    51.9

Error: subj:tempo
           Df Sum Sq Mean Sq F value  Pr(>F)
tempo       1  275.0  275.01   9.704 0.00268 **
tempo:trat  2  307.3  153.66   5.422 0.00650 **
Residuals  69 1955.4   28.34
```